

Stats 2 - June 2007

- ① H_0 : No association between outcome & treatment (Independent)
 H_1 : Association between outcome & treatment (non-independent)
 2 by 2, so need to use Yates's correction

Observed
Expected

	P	D	
Imp	20	46	(66)
Not	55	29	(84)
	(75)	(75)	(150)

Expected

	P	D
Imp	33	33
Not	42	42

$$X^2 = \frac{(10 - |E| - 0.5)^2}{E}$$

$$|10 - |E| - 0.5| = 12.5$$

	P	D
Imp	4.7348	4.7348
Not	3.7202	3.7202

$$X^2 = 16.91 \quad (\text{Test Statistic})$$

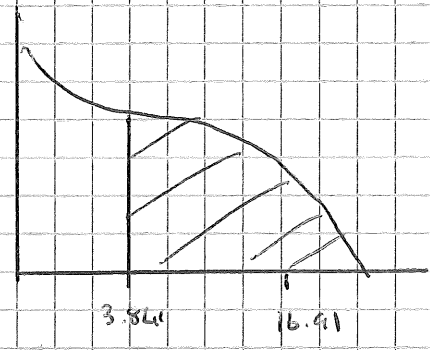
CRITICAL VALUE: $\nu = (2-1)(2-1) = 1$

$$X^2_{5\%}(1) = 3.841$$

$$16.91 > 3.841$$

Reject H_0

Evidence at 5% level suggests outcome (condition) may be dependent on treatment received



② a) i) $X \sim \text{Po}(3.5)$

$$P(X=3) = \frac{e^{-3.5} \times (3.5)^3}{3!} = 0.216$$

ii) $Y \sim \text{Po}(6)$

$$P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - 0.2851 \quad (\text{from tables}) = 0.7149$$

b) i) $T \sim P_0(9.5)$.

ii) $P(7 \leq T \leq 10) = P(T \leq 10) - P(T \leq 6)$
 $= 0.6453 - 0.1649 = 0.4804$

iii) $p = (0.4804)^3 = 0.11086$

③ $H_0: \mu = 36$

$H_1: \mu < 36$

$n = 50$

$\bar{x} = 1730/50 = 34.6$

$\sum x = 1730$

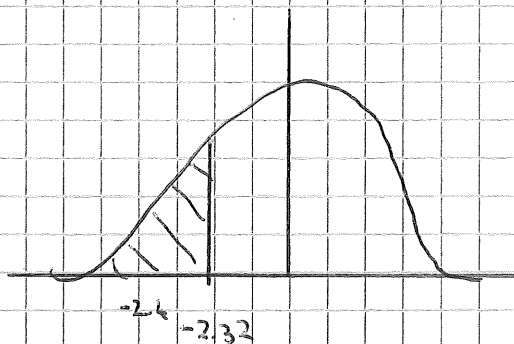
$s^2 = 784/49 = 16$

$\sum (x - \bar{x})^2 = 784$

$s = 4$

Test Statistic: $\frac{34.6 - 36}{4/\sqrt{50}} = -2.4748...$

Critical Value: $Z, 1\%, 1 \text{ tailed test} = -2.3263$



$-2.47 < -2.32$

Reject H_0

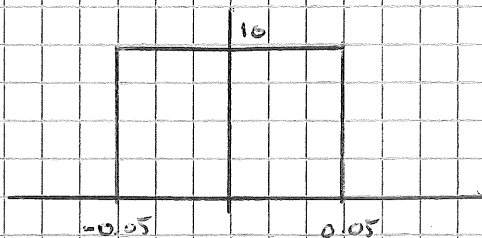
Sufficient evidence at 1% level to suggest average number of puts has decreased.

④ a) Nearest $\frac{1}{10}$ in m has bounds -0.05 to 0.05

For rectangular distribution $f(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases}$

$-0.05 \leq x \leq 0.05$

$\rightarrow \frac{1}{b-a} = \frac{1}{0.05 - (-0.05)} = 10$



Area = $10 \times \frac{1}{10} = 1$

b) From diagram = $0.03 \times 10 = 0.3$

c) Mean = $\frac{0.05 + (-0.05)}{2} = 0$

Variance = $\frac{1}{12} (0.05 - (-0.05))^2 = \frac{1}{12} (0.01)$

$\therefore s.d = \sqrt{\frac{1}{12} (0.01)} = 0.02886...$

5) a) From calculator:

$$n = 10$$

$$\bar{x} = 35.6$$

$$\sum x = 356$$

$$s = 6.1860..$$

$$\sum x^2 = 13,018$$

$$s^2 = 38.2666..$$

99% CI. No σ^2 , so need t distribution,

$$v = 10 - 1 = 9 \rightarrow t = 3.250 \quad (\text{look up } 0.995)$$

$$99\% \text{ CI} = 35.6 \pm 3.250 \times \frac{6.1860}{\sqrt{10}}$$

$$= 35.6 \pm 6.357..$$

$$= (29.2, 42.0)$$

b) Confidence interval includes 30mph

BUT 8/10 cars in sample exceed it

\therefore speed limit not being stuck to by most motorists

$$\begin{aligned} \text{(b) a) i) } E(1/x) &= \int_0^1 1/x \cdot f(x) \, dx \\ &= \int_0^1 1/x (3x^2) \, dx = 3 \int_0^1 dx \, dx \\ &= [3x^2/2]_0^1 = 3/2 \text{ or } 1.5 \end{aligned}$$

$$\begin{aligned} \text{ii) } E(1/x^2) &= \int_0^1 1/x^2 \cdot f(x) \, dx \\ &= \int_0^1 1/x^2 (3x^2) \, dx = \int_0^1 3 \, dx \\ &= [3x]_0^1 = 3 \end{aligned}$$

$$\begin{aligned} \text{Var}(1/x) &= E(1/x^2) - [E(1/x)]^2 \\ &= 3 - 1.5^2 = 0.75 \end{aligned}$$

$$\begin{aligned} \text{b) MEAN: } E\left(\frac{5+2(x)}{x}\right) &= E\left(\frac{5}{x}\right) + E(2) \\ &= 5E\left(\frac{1}{x}\right) + 2 \\ &= 5(1.5) + 2 = 9.5 \end{aligned}$$

VARIANCE:
$$\text{Var} \left(\frac{5 + 2x}{x} \right) = \text{Var} \left(\frac{5}{x} \right) + \text{Var} (2)$$

$$= 5^2 \text{Var} \left(\frac{1}{x} \right) + 0$$

$$= 25 \times 0.75 = 18.75$$

7) a) i)

x	4	-1
$P(X=x)$	$\frac{1}{5}$	$\frac{4}{5}$

ii) $E(X) = 4 \left(\frac{1}{5} \right) + (-1) \left(\frac{4}{5} \right) = 0$

b)

x	4	-1
$P(X=x)$	$\frac{1}{3}$	$\frac{2}{3}$

$E(x) = 4 \left(\frac{1}{3} \right) + (-1) \left(\frac{2}{3} \right) = \frac{2}{3}$

out of 24 = $24 \times \frac{2}{3} = 16$

24 E(x)

8) a) $H_0: \mu = 230$

$H_1: \mu \neq 230$ (2 tailed)

No σ^2 , so must use t -distribution, $v = 8 - 1 = 7$

From calculator:

$n = 8$

$\bar{x} = 225.25$

$\sum x = 1802$

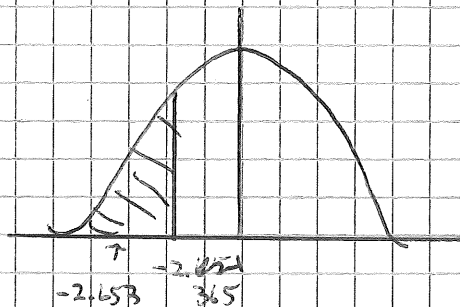
$s = 5.0638..$

$\sum x^2 = 406,090$

$s^2 = 25.642..$

TEST STATISTIC:
$$\frac{225.25 - 230}{\frac{5.0638}{\sqrt{8}}} = -2.6531..$$

CRITICAL VALUE: $t_{\frac{5\%}{2}} (7)$, 2 tailed test = ± 2.365



$-2.653 < -2.365$

\therefore Reject H_0

Evidence at 5% level suggests that average weight in jar is not 230g

b) Rejected H_0 when H_0 was true = Type 1 Error.